Introduction

Central banks hold foreign reserves to facilitate international trade and as a mean of financing exchange rate interventions in the foreign exchange market. Reserves can therefore be thought of as a buffer stock, held for precautionary purposes.[1] Central banks' *total* demand for reserves should therefore depend on factors such as the magnitude of foreign trade, the marginal propensity to import, the opportunity cost of holding reserves, current account volatility and exchange rate arrangements.[2] The *relative* demand for reserves, i.e. the composition of reserves, depends however on such factors as the real rate of return on currencies and the volatility of these returns, the central banks' risk aversion, transaction costs and, possibly, additional non-economical factors and institutional arrangements.

The purpose of this paper is to study the relative demand of central banks for currency assets, taking the Central Bank of Iceland as an example.[3] The paper is divided into five sections, plus appendices. The first section describes the historical development of currency compositions of central banks and discusses the main determinants of relative currency demand. The second section analyses in more detail the mean-variance and the transaction cost models, which are the two main models for explaining the currency composition of reserves. In the third section we calculate optimal portfolios for the Central Bank of Iceland in the period 1987 to 1993, using the mean-variance model, and compare the resulting portfolios with actual data. The fourth section contains estimation of demand functions for relative currency demand, using two alternative modelling procedures. The fifth section concludes.

1. The Currency Composition of Foreign Reserves

1.1. Historical Development

Looking at the currency composition of reserves of all countries we see that the share of the U.S. dollar has gradually been decreasing, although the dollar is still by far the most important reserve asset. However, these changes in the composition of foreign reserves have encompassed somewhat contrasting behaviour on the part of industrial

---

[1] This is the standard assumption in this literature. It seems a highly plausible assumption for any central bank, especially for small central banks such as the Central Bank of Iceland. See, for example, Ben-Bassat (1980), Rikkonen (1989) and Dellas and Yoo (1991).

[2] There are number of studies available on the total demand for reserves. See, for example, Frenkel and Jovanovic (1981), Heller and Kahn (1978), Landell-Mills (1989), Lehto (1994), Lizondo and Mathieson (1987) and Roger (1993).

[3] Even though total and relative demand for reserves are in some ways linked, it is commonly assumed that these decisions are independent. This tradition is followed here.

and developing countries. For industrial countries, the dollar share has decreased, but for developing countries there is actually a small increase in the dollar share. The drop in the dollar share has largely been matched by an increase in the share of the German mark. Comparison of currency compositions of industrial and developing countries also shows that the currency composition in the developing countries is more stable than in the industrial countries.[4] This can largely be explained by fluctuations in the currency compositions of industrial countries, reflecting the effects of interventions related to exchange rate movement rather than fundamental shifts in portfolio preferences.

Table 1. The currency composition of foreign reserves of the Central Bank of Iceland in per cent 1987-1993

Countries	*Currency*	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*	*1993*	*Average*
Iceland	USD	43.2	46.5	45.6	42.8	41.6	32.7	36.1	41.2
	DEM	19.5	24.2	23.4	21.5	23.2	29.0	30.5	24.5
	GBP	0.4	0.8	4.5	7.2	1.5	4.8	2.6	3.1
	JPY	15.5	16.6	14.6	12.2	13.1	9.8	9.5	13.1
	Others	21.4	11.9	11.9	16.3	20.6	23.7	21.3	18.1
Industrial countries	USD	71.4	67.7	59.6	55.9	55.8	62.8	60.7	62.0
	DEM	15.9	17.3	22.5	21.9	20.0	16.3	19.7	19.1
	GBP	1.1	1.5	1.4	1.9	2.0	2.5	2.7	1.9
	JPY	7.1	7.0	8.1	9.6	10.4	8.0	8.7	8.4
	Others	4.5	6.5	8.4	10.7	11.8	10.4	8.2	8.6
Developing countries	USD	59.9	57.4	61.3	60.7	62.7	63.8	62.3	61.2
	DEM	10.8	11.4	10.9	11.6	10.8	10.8	11.4	11.1
	GBP	5.1	5.5	5.5	6.4	6.0	4.5	4.3	5.3
	JPY	8.5	9.2	6.8	7.3	7.6	9.1	9.5	8.3
	Others	15.7	16.5	15.5	14.0	12.9	11.8	12.5	14.1
All countries	USD	67.8	64.6	60.2	57.5	58.4	63.2	61.4	61.9
	DEM	14.3	15.5	18.8	18.6	16.5	14.1	16.1	16.3
	GBP	2.4	2.7	2.7	3.4	3.6	3.3	3.4	3.1
	JPY	7.5	7.7	7.7	8.8	9.4	8.5	9.0	8.4
	Others	8.0	9.5	10.6	11.7	12.1	10.9	10.1	10.3

Source: Central Bank of Iceland and IMF: *Annual Report*, various publications.

The currency composition of the foreign reserves of the Central Bank of Iceland has generally developed in a similar fashion to those of other industrial countries. The share of the dollar has been decreasing, matched by an increase in the share of the mark. The relative share of the dollar has, however, always been smaller than in other industrial countries and the Japanese yen share larger, although it has also decreased recently. It can also be seen from the table that the volatility of currency shares of the Central Bank of Iceland is greater than for other countries (the ratio of

[4]The ratio of standard deviation to the mean share of currencies is on average 13 per cent for the developing countries but 25 per cent for the industrial countries.

standard deviation to mean currency share is on average 38 per cent for Iceland). This can probably be explained by large foreign loans that can change the currency composition quite substantially.

1.2. The Determinants of Relative Currency Demand

There are mainly two views concerning which factors are important in determining relative currency demands of central banks. The first is based on the mean-variance model from portfolio theory, which states that central banks choose the composition of reserves that minimises the risk of the reserves for a given return on the reserves.[5]

The latter view argues that central banks have broader objectives than portfolio optimisation. This view argues that transaction costs are a key factor in determining currency shares, leading to the so-called "vehicle currency motive", which states that the currencies with the largest portfolio shares are those who are most important for financing foreign trade and foreign debt obligations. We call this motive the transaction cost model. These alternative models are discussed in more detail in the next section.

2. Theoretical Framework

2.1. The Mean-Variance Model

The mean-variance model was developed to describe how private investors could diversify their asset holdings so as to minimise their portfolio risk in the seminal papers by Markowitz (1952) and Tobin (1958). Here we use this model to describe the optimal composition of foreign reserves. In this case the central bank chooses the relative currency weights so that the total risk on the portfolio is minimised for a given return on reserves.[6]

Let m_{it} be the *ex post* real rate of return on currency *i*, $i = 1,...,G$, from time *t* to time $t+1$. m_{it} is unknown at time *t*, but we can denote its conditional mean as m_{it}, its conditional variance as s_{iit} and the conditional covariance between currency returns *i* and *j* as s_{ijt}. Expectations are conditional on the information set Q_t, which is known at time *t*, i.e. $E_t(m_{it}) \equiv E(m_{it}|Q_t) = m_{it}$, where $E(\ )$ is the mathematical expectations operator.

[5]This approach is being adopted by an increasing number of central banks. See, for example, Allen (1994) and Downes (1989).

[6]As foreign exchange markets are assumed to be efficient, small central banks can be viewed as price takers who cannot affect exchange rates. They can however reduce their exposition to currency risk by diversifying their reserves as the following results indicate.

The central bank's problem is to choose a portfolio of currencies in each period that minimises the variance of the portfolio for a given expected real return:

$$(1) \qquad \min_{w_{it}} s_{Rt}^{2} = \sum_{i=1}^{G} \sum_{j=1}^{G} w_{it} w_{jt} s_{ijt} ,$$

so that:

$$(2) \qquad m_{Rt} = \sum_{i=1}^{G} w_{it} m_{it} ; \qquad \sum_{i=1}^{G} w_{it} = 1; \qquad w_{it} \geq 0; \quad i = 1, \dots, G ,$$

where s_{Rt}^{2} is the variance of the portfolio, m_{Rt} is the expected real return on that portfolio and w_{it} is the relative share of currency *i* in the portfolio. This problem does not have a unique solution, as the optimal solution depends on the risk aversion of the central bank. The set of portfolios that solve this problem are described by the efficient frontier. To get a unique solution, the preferences of the central bank must be explicitly formulated with an expected utility curve. We assume that expected utility is positively related to expected real return on the portfolio but negatively related to the portfolio risk. A simple example of such an expected utility curve is:[7]

$$(3) \qquad U(m_{Rt}, s_{Rt}^{2}) = m_{Rt} - r s_{Rt}^{2} / 2 ,$$

where r is the Arrow-Pratt measure of risk aversion, which also measures the market price of risk.[8]

When the expected utility curve has been formulated, a single solution to the portfolio problem can be found by maximising (3) with respect to m_{Rt}, for a given s_{Rt}^{2} that lies on the efficient frontier. The solution to this problem can be described as:[9]

$$(4) \qquad W_{t}^{**} = W_{t}^{*} + r^{-1} W_{t}^{-1} E_{t}(X_{t}) ,$$

[7]This expected utility function can be derived from a concave utility function, *V*(*R*), where *R* are reserves and $-V''/V' = r$ is the Arrow-Pratt measure of risk aversion. If we assume a density function for normal distributions, $f(R; m_R, s_R^2)$, expected utility can be found as $E(V(R)) = \int_{-\infty}^{\infty} f(R; m_R, s_R^2) V(R) dR = U(m_R, s_R^2)$ if V satisfies $|V(R)| \leq ae^{-bR^2}$ for any *a*, *b* > 0. See Sargent (1987).

[8]The optimal portfolio composition can be derived without explicitly formulating the preferences of the central bank if a risk free investment is available to the bank, according to the CAPM. The problem with this approach is that risk free investments for central banks are generally not available. This is why we do not proceed in that direction. See Roger (1993) and Ben-Bassat (1980).

[9]See appendix B for a derivation of (4).

where W_t^* is a vector of currency share, describing the minimum variance portfolio, W_t is the variance-covariance matrix of currency returns and $E_t(X_t)$ is a vector of expected returns on currencies in excess of the return on the minimum variance portfolio, conditional on the information set Q_t. From equation (4) we see that the optimal portfolio can be divided into two portfolios: the minimum variance portfolio, W_t^*, and a speculative portfolio, $W_t^{-1}E_t(X_t)$.[10] r is therefore the relative weight that determines the corresponding weights of the minimum variance and the speculative portfolios in the total portfolio.

2.2 Calculating Real Returns on Currencies

Levy and Sarnat (1975) have shown that if investors face different international prices (because of non-traded goods, tariffs, taxes, etc.), optimal portfolios will depend on consumption patterns. Consequently, the problem for central banks is more complicated than that of private investors. For private investors it is usually assumed that the portfolio is valued in the currency of his or her consumption, but for central banks the choice of the consumption currency is not so obvious. As mentioned in the introduction, it is usually assumed that foreign reserves are, *inter alia*, meant for financing imports. Reserves should therefore be valued by their real purchasing power and real returns calculated by deflating nominal returns with an import price index.[11]

Expected real rate of return is therefore calculated as:

$$(5) \qquad m_{it} = \frac{(1+r_{it})(E_t(e_{it+1})/e_{it})}{(E_t(P_{t+1})/P_t)} - 1,$$

where r_{it} is the nominal return on bonds denominated in currency *i*, between time *t* and time *t*+1, e_{it} is the exchange rate of the domestic currency per foreign currency, $P_t = \prod_{i=1}^{N}(e_{it}P_{it}^*)^{a_{it}}$ is the import price index, P_{it}^* is the import price level in country *i*, a_{it} is the weight of currency *i* and *N* is the number of currencies in the import price index.[12] If we accept that reserves are ultimately used to finance imports, theoretically

---

[10]It should be noted, that when the uncovered interest rate parity (UIP) holds the efficient frontier collapses to a single point, that point being the minimum variance portfolio. The intuition for this is that when UIP holds, all expected returns are equal.

[11]The import price index is still the appropriate deflator for real returns, even though reserves are never used for financing of imports, provided we are interested in the real value of the reserves in the purchasing power sense. See Rikkonen (1989).

[12]An alternative investment scheme would be to preserve the nominal value of reserves. In this case we would usually measure the reserves in terms of a vehicle currency, which is most often the dollar. Ben-Bassat (1980) calculates the optimal currency composition for Israel using dollar denominated returns. His results show that in the minimum variance portfolio the optimal share of the dollar is 96 per cent. A similar model for Iceland gives a minimum variance portfolio of 98 per cent

it is more appropriate to use currency weights than import weights. However, due to data availability most studies have used import weights as a proxy for currency weights.[13] This is, however, not a good proxy, especially not for the dollar, as the dollar is used more extensively in international trade than is captured by bilateral trade weights with the United States. Dellas and Yoo (1991) were the first to use currency weighted import prices to calculate real rates of return and their work shows a significant difference in results of the mean-variance model when the two alternative measures of weights are used. Most significantly, is an increase in the optimal weight of the dollar when currency weights are used. As currency weights are also available in our case, we base our calculations of real returns for the Central Bank of Iceland on those weights.[14]

To calculate mean returns we proceed in two ways. First we calculate the *ex post* real returns, m_{it}, which is equivalent to assuming perfect foresight on behalf of the central bank.[15] For comparison, we also calculate the *ex ante* real returns, by assuming that exchange rate returns follow a random walk, and forecast import prices with a parsimonious ARIMA model.[16] These two alternatives measures of real returns lead to quite substantial differences in the optimal composition of foreign reserves.

2.3. Theoretical Assumptions of the Mean-Variance Model

There are two critical assumptions of the mean-variance model worth mentioning. The first one concernes the stability of the variance-covariance matrix of currency returns. This is a common assumption in the finance literature, but has increasingly been questioned, especially for high frequency data, such as daily returns. Recent empirical findings indicate that variances of high frequency returns follow an autoregressive process. According to these results it would be incorrect to assume a constant W_t, leading to suboptimal currency holdings. One approach that takes account of this phenomenon is the ARCH model (and its generalisations), which directly uses the autoregressive properties in variances of asset returns.[17]

---

dollar and 2 per cent mark. We also calculated the optimal currency shares when the vehicle currency was measured as the exchange rate index: 76 per cent ECU, 18 per cent dollar and 6 per cent yen. These results gave yet another optimal composition: 33 per cent dollar, 38 per cent mark, 28 per cent pound sterling and 1 per cent yen.

[13]See, for example, Ben-Bassat (1980), Rikkonen (1989) and Horii (1986).

[14]Further description of the data is in Appendix A.

[15]Most studies are based on *ex post* returns.

[16]The ARIMA model provides a better out of sample forecast of the log-differenced import price level than a simple random walk model. See Appendix C.

[17]See Engle (1982) and Bollerslev, Engle and Nelson (1993).

In Table 2 we estimate returns with ARMA models to test for homoscedastistic residuals in krona denominated returns. We also compare the results of using real returns on currencies (deflated with import prices) with nominal exchange rate returns. The ARCH tests for autocorrelation up to 5th degree indicate that there is no significant autocorrelation in the conditional variances of krona denomintaed real returns, whereas there is evidence in favour of such effects in krona denominated nominal exchange rate returns, in the case of the mark and the yen. We can thus reject the hypothesis of time-varying variances in krona denominated real returns in favour of the hypothesis of constant variances. For exchange rate returns this result does, however, not hold. This differs from the results of Papaioannou and Temel (1994). They use an ARCH model, as is done here, and find that dollar denominated returns on currencies can be described by an ARCH realisation. These effects do not show up in the krona denominated real returns used here. As the results in Table 2 indicate, the use of the import price level to calculate real return on currencies seems to filter the ARCH effect out of the data.

Table 2. Statistical diagnostics of returns

Monthly data 1980-1993

	$\ln(1+m_{it})$				$\ln(e_{it+1}/e_{it})$			
	USD	DEM	GBP	JPY	USD	DEM	GBP	JPY
Skewness	0.090	0.005	-0.261	0.301	0.660	1.131	0.915	0.891
Kurtosis	2.128	2.276	3.722	2.650	2.721	3.863	4.170	3.747
Bera-Jarque	5.552	3.675	5.553	3.398	12.761**	41.052**	33.020**	26.112**
ARCH(1)	0.037	0.417	0.564	0.461	1.396	2.557	2.142	3.435
ARCH(5)	3.159	4.623	6.164	4.614	4.523	14.266*	5.049	13.363*
$CHOW_{83:3}$	3.096*	3.108*	0.423	1.344	5.426**	2.818*	1.554	3.415**
$CHOW_{90:1}$	3.150*	1.901	0.733	3.694**	2.527*	0.460	2.024	1.963
HF(12)	17.506	12.532	5.070	15.889	15.688	9.978	10.656	12.162
Model for returns	ARMA (1,2)	ARMA (1,3)	ARMA (1,2)	ARMA (3,3)	ARMA (2,4)	ARMA (1,3)	ARMA (2,3)	ARMA (3,3)

Note: * (**) indicates statistical significance at the 5 per cent (1 per cent) level. The number of observations is 168. The Bera-Jarque test for normality is distributed as $c^2(2)$ under the null hypothesis of normality. ARCH(r) is distributed as $c^2(r)$ under the null hypothesis of non-significant authocorrelation in the conditional variance up to order r. The Chow test for structural break is distributed as $F(n_1, n_2)$ under the null of no structural change. HF(m) is the Hendry out of sample forecast test for a m period horizon, asymptotically distributed as $c^2(m)$ under the null of equal variances in the estimation and the forecasting periods.

Although time-varying variances in real returns can be rejected it could be that the hypothesis of a constant variance-covariance matrix is rejected due to structural changes in the estimation period.[18] To test for this we constructed several Chow tests. The residuals from the above ARMA models indicate that such as break could have occured in early 1983. A formal test for a structural break in 1983:3 supports this conjecture. This suggests that the residuals from the ARMA model for real and nominal returns are heteroscedastistic, although this is rejected at the 1 per cent level for real returns. But as we are only calculating the optimal currency portfolios for the period 1987 to 1993, this is not of major importance. More important is to test for a structural break in the period 1987 to 1993. To do this we simply constructed a Chow test for a structural break in the middle of the period, 1990:1. The results from this test indicate that the real returns of the dollar and the yen do not have constant variances.

Finally, we calculated the Hendry forecast test for the hypothesis of constant variances in the estimation period up to 1993 and the 12 month forecast period, 1993:1 to 1993:12. On the basis of this test we cannot reject the hypothesis of equal variances in the estimation and forecast periods.

Concluding from this analysis, there is no evidence of time-varying variances in real returns on currencies, although such effects are significant in nominal returns. This indicates that the currency shares from the mean-variance model in section 3 are optimal, in the sense of efficient use of available information. There is, however, some evidence of heteroscedasticity in real returns due to structural breaks in the data, suggesting shifts in the variance-covariance matrix in the period 1987 to 1993. This indicates that the currency shares from the mean-variance model are not necessarily stable when different time periods are used, possibly resulting in poor out of sample performance of the portfolio.

Another implicit assumption of the mean-variance model is that the rate of returns and risk of currencies can be adequately described by the first two moments of the probability distribution, i.e. the mean and the variance. For normally distributed returns this would be sufficient, but non-normally distributed returns call for the use of other moments of the probability distribution, such as the third and fourth moment, i.e. the skewness and the kurtosis. Most economists accept that exchange rate returns, as most other asset returns, are not normally distributed but are leptokurtic, that is, the distribution has fatter tails and higher peaks than the normal distribution.[19] In Table 2 we test whether the assumption of normality is rejected by the data. The Bera-Jarque normality test indicates that real returns on currencies are normally distributed, whereas the normality of exchange rate returns is strongly rejected.

[18]It should be mentioned that the ARCH test has power against a variety of heteroscedasticity alternatives.

[19]See, for example, de Vries (1994).

2.4. The Transaction Cost Model

Dooley (1987) and Dooley, Lizondo and Mathieson (1989) argue that a clear distinction must be made between the currency composition of gross and net reserves (net of foreign liabilities). Dooley, Lizondo and Mathieson show that the currency composition of net reserves is a function of returns and risk, whereas the composition of gross reserves is related to the country's exchange rate arrangement and the currency composition of foreign trade and liabilities.

They postulate an expected utility function that is negatively related to expected transaction costs:

$$(6) \qquad U(m_{Rt}, s^2_{Rt}, C_t) = m_{Rt} - r\, s^2_{Rt} / 2 - C_t,$$

where C_t is the expected transaction cost in period *t*, representing the cost of using the foreign exchange market for currency substitution and possible emergency borrowing costs.

If H_t denotes the currency shares of net reserves, Dooley, Lizondo and Mathieson (1989) show that:

$$(7) \qquad H_t^* = H(m_{Rt}, s_{Rt}); \quad H_m > 0 \quad H_s < 0.$$

For, currency shares in gross reserves, however, we can postulate the following functional relationship:

$$(8) \qquad W_t^* = W(T_t, D_t), \quad W_T, W_D > 0,$$

where T_t is the share of currencies in foreign trade and D_t is the share of currencies in foreign debt in period *t*.

From equation (7) it can be seen that the currency composition of net reserves is determined by expected real returns on currencies and currency risk, whereas, from equation (8), currency shares in gross reserves are independent of these variables. Currency shares in gross reserves are determined by the transaction cost model, represented here by currency shares in foreign trade and debt servicing payments.

The problem with this approach, as Dooley (1987) points out, is that the institutional arrangements in most countries may well prevent portfolio optimisation of net reserves, the reason being that, in most cases, different institutions are in charge of gross reserves (usually the central bank) and foreign loans (often the ministry of finance). Further, the ability to manage currency positions on the liability side may be much more constrained as liabilities typically have longer maturity. Another reason for questioning this approach is that transaction costs in foreign exchange markets are neglectable, especially for the most important currencies.[20] Therefore, we argue that the mean-variance model is still the appropriate model for currency compositions of gross returns.

Table 3 shows the currency composition of foreign debt and foreign trade for Iceland in the period 1987 to 1993. It is clear from the table that the currency composition of foreign debt is very similar on average to the composition of reserves in this period (see Table 1). The same applies to foreign trade, although the share of the mark and yen are relatively smaller in foreign trade and the share of the British pound larger.

Table 3. Currency composition of foreign trade and debt for Iceland in per cent 1987-1993

Table 3.*a*. Currency composition of foreign debt

Currency	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*	*1993*	*Average*
USD	42.0	44.9	47.2	50.1	54.6	51.9	48.8	48.5
DEM	16.7	18.3	22.8	22.5	20.3	18.7	17.6	19.6
GBP	5.3	4.8	3.7	3.8	3.4	2.7	6.7	4.3
JPY	15.0	13.3	10.0	8.7	8.9	11.7	11.2	11.3
Others	21.0	18.7	16.3	14.9	12.8	15.0	15.7	16.3

Table 3.*b*. Currency composition of foreign trade

Currency	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*	*1993*	*Average*
USD	47.5	50.7	52.9	47.3	44.7	38.0	38.5	45.6
DEM	14.5	11.8	12.3	13.0	12.2	14.2	11.9	12.8
GBP	9.9	10.0	8.8	11.8	10.0	9.8	12.1	10.3
JPY	3.9	3.5	3.9	3.3	4.5	5.1	8.1	4.6
Others	24.2	24.0	22.1	24.6	28.6	29.9	29.4	26.7

Source: Central Bank of Iceland.

3. The Optimal Composition of Foreign Reserves

In this section we present results on the optimal composition of reserves for the Central Bank of Iceland, using the mean-variance model, for the period 1987 to 1993.[21] We analyse four currencies, the dollar, the mark, the pound, and the yen.[22]

[20]Formally, transaction costs should lie between the spread between bid and ask prices, on the one hand, and deviations from cross exchange rates, on the other. Most studies indicate that this cost is neglectable. See discussions in Pétursson and Helgason (1994).

[21]The optimal portfolios for the period 1980 to 1986 was also calculated and are available from the author upon request.

[22]Originally both the Dutch guilder and the Swiss frank were also analysed. It was however decided to drop the guilder from the portfolio, due to a very high correlation with the mark (about 0.99). The Swiss frank had a zero share in the optimal composition in all cases and is therefore not reported here.

These four currencies represent more than 80 per cent of the reserves on average in this period.[23]

Two alternative compositions are calculated, using different assumptions for the Central Bank's information set, Q_t. First, we simply assume perfect foresight and calculate the *ex post* optimal shares. Secondly, we calculate the *ex ante* optimal shares, according to a simple forecasting model for real returns. This model assumes that currencies are described as random walk processes, which indicates that the best forecasts for future exchange rates are current exchange rates. Recent research clearly suggests that this simple forecasting scheme has better success than more complicated structural models in forecasting out of sample exchange rates in low frequency data.[24] To forecast the import price level, a simple ARIMA model for the period 1980 to 1986 is estimated and used to forecast real returns for the period 1987 to 1993. This is described in more detail in Appendix C.

The optimal currency shares are shown in Table 4. Four different compositions are shown, using different values for the Central Bank's risk aversion, r. The first composition is the minimum-variance composition. The second composition is calculated assuming that the Central Bank assigns equal weights to the minimum variance and the speculative portfolios. The third composition assumes a relatively risky investment scheme and the last composition is the speculative portfolio.

Table 4. Optimal currency compositions for the Central Bank of Iceland 1987-1993

	Ex post real return				*Ex ante* real return			
Risk aversion	∞	1.0	0.2	0.0	∞	1.0	0.2	0.0
Real return	4.902	4.961	5.490	8.321	2.626	6.678	6.678	6.678
Standard deviation	2.963	2.972	3.829	19.242	1.936	2.590	2.590	2.590
USD	48%	47%	40%	0%	62%	0%	0%	0%
DEM	28%	28%	20%	0%	32%	0%	0%	0%
GBP	24%	24%	30%	0%	0%	100%	100%	100%
JPY	0%	1%	10%	100%	6%	0%	0%	0%
Total	100%	100%	100%	100%	100%	100%	100%	100%

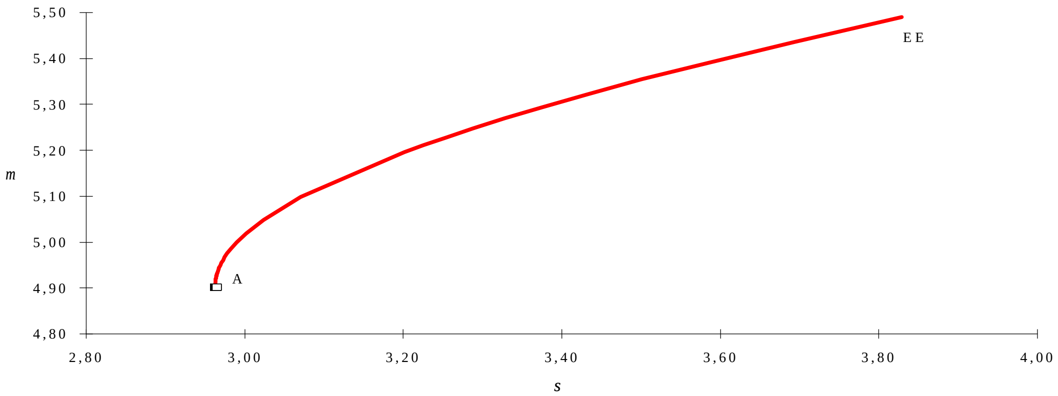
Using *ex post* real returns indicates that the optimal composition is not sensitive to the assumption of risk aversion, except when we assume very risky portfolios such as the speculative portfolio. The optimal composition using *ex ante* real returns is, on the other hand, very sensitive to the Central Bank's level of risk aversion.

[23] As there are only four currencies used here, we proceed by truncating the actual shares so that they sum to 1 to ease all comparison.

[24] See, for example, de Vries (1994).

It can also be seen that, using *ex post* returns, higher return can be attained by increasing the portfolio risk only slightly, which indicates that the efficient frontier is quite steep near the minimum variance portfolio. This can be seen from Figure 1 which shows the efficient frontier using *ex post* real returns.

Figure 1. The Efficient frontier for the Central Bank of Iceland 1987-1993
Ex post real return



The difference in the optimal *ex post* minimum variance composition and the *ex ante* minimum variance composition lies mainly in the share of the pound sterling. The optimal *ex post* minimum variance share of the pound is quite high whereas the *ex ante* share of the pound is zero. The more risky *ex ante* portfolios, however, only include the pound. The explanations for these different results can be seen in Table 5, which shows the return and standard deviation of each currency and the correlations of returns under the two alternative assumptions.

Table 5. Returns and correlations 1987-1993

Table 5.*a*. *Ex post* real return

	Return	Standard deviation	USD	DEM	GBP	JPY
USD	3.47	13.39	1.000	-0.729	-0.747	-0.034
DEM	5.06	13.68		1.000	0.364	0.076
GBP	7.61	16.59			1.000	0.091
JPY	8.32	19.24				1.000

Table 5.*b*. *Ex ante* real return

	Return	Standard deviation	USD	DEM	GBP	JPY
USD	2.51	2.28	1.000	0.154	0.729	0.606
DEM	3.11	2.97		1.000	0.407	0.748
GBP	6.68	2.59			1.000	0.738
JPY	1.27	2.21				1.000

In Table 5.*a* it can be seen that the dollar and mark are the least risky assets and that the correlation between the dollar, on one hand, and mark and pound, on the other hand, is high and negative. At the same time there is little correlation between the mark and the pound. This clearly indicates that it is desirable to use these three currencies in the reserves to reduce the portfolio risk. For more risky investments the share of the yen should clearly rise as the yen has the highest return of all the currencies. In Table 5.*b* completely different patterns are seen. Now, the yen has the lowest return and the correlations are positive between returns on all currencies. There is however a small correlation between the mark and the dollar, suggesting the use of both these currencies in the optimal minimum variance portfolio. As the portfolio risk increases, the share of the pound increases as the pound has the highest return.

A comparison of *ex post* optimal currency shares with actual, average currency shares for Iceland in Table 1 shows very similar currency shares of the dollar and the mark, but a significant difference in the pound and the yen shares. As the yen is a relatively risky asset in this period, this indicates that the actual, average portfolio was riskier than the *ex post* optimal portfolio. The real return on the average portfolio was 4.89 per cent, with a standard deviation of 5.27. This suggests that the mean-variance model is not sufficient to explain actual relative currency demand of the Central Bank of Iceland.

4. Estimating Relative Currency Demands

Comparing actual currency shares with those predicted by the mean-variance model indicates that other factors are instrumental in explaining the currency composition of the Central Bank of Iceland's reserves. It is however interesting to test the mean-variance model more formally and, simultaneously, test the transaction cost model. In this section we do this using two approaches. The first one is simply to estimate a model where the real rate of return on currencies, currency risk, currency share in foreign trade and currency share in foreign debt explain the composition of foreign reserves of the Central Bank of Iceland. The second method uses an approach suggested by Frankel (1982), which directly tests theoretical restrictions implied by the mean-variance model. In both cases data for the period 1980 to 1993 is used.

4.1. The Mean-Variance Model Versus the Transaction Cost Model

In the empirical model that is estimated in this section, we try to take into account both the mean-variance model and the transaction cost model. We hope to be able to say which model has better explanatory power in explaining the relative currency demand of the Central Bank of Iceland. Formally the model can be described as:

$$(9) \qquad W = W(m, s, T, D); \quad W_m, W_T, W_D > 0, \quad W_s < 0.$$

The results shown here are calculated using *ex post* real returns. The main results did not change when *ex ante* real returns were used.

The model uses annual data from 1980 to 1993 for the dollar, the mark, the pound and the yen.[25] We have therefore only 14 observations for each currency, which are too few to estimate a significant empirical model. We therefore assume that return, risk, the currency share in trade and debt all affect each currency in the same way, which seems not so economically unreasonable. This allows us to pool together time series and cross-section data to increase the number of observations to 56 (the results for each currency are shown in table 9 for further information). To allow for currency-dependent effects, a currency-specific constant for each currency is developed.

The estimated model is a linear approximation of (9):

$$(10) \qquad W = b_{USD} + b_{DEM} + b_{GBP} + b_{JPY} + b_1 m + b_2 s + b_3 T + b_4 D.$$

We first estimate the model using the OLS rule. This method is simple to implement but rests on rather strict assumptions. These assumptions are that of no contemporary correlations between currency shares and constant volatility across currency shares. The second assumption can possibly be accepted but the first one is obviously inappropriate in the model to be estimated here. This indicates that the standard errors of the OLS parameter estimates will be incorrect which could lead to inappropriate inference rules. Therefore, we estimate the model again using the GLS rule, which allows for contemporary correlation across currency shares and different volatility in those shares. Dooley, Lizondo and Matthieson (1989) try to explain currency shares in reserves with the transaction cost model. They point out that currency shares can only take values in the interval (0,1) and the use of least squares rules (LS) would therefore be inappropriate, as LS methods assume that currency shares can take on all values on the real line. This would therefore lead to biased and inefficient estimators. They use the Tobit estimation technique instead to estimate currency shares. For comparison they also estimate the model using OLS. The comparison indicates that the OLS technique underestimates the explanatory power of the transaction cost model. This is interesting since all empirical results of the transaction cost model have been based on OLS estimations.

It is possible to verify whether the use of LS techniques leads to a serious bias in the estimated coefficients and standard errors by testing whether the LS residuals are normally distributed. Failure to reject normality indicates that the problem of using LS

[25] Data on higher frequency is not available for the currency composition of foreign trade and debt.

methods is not so serious in our case. For our model the Bera-Jarque statistic is 0.85, that is well below all commonly used statistical levels. We can therefore assume that the residuals from the LS estimations are normally distributed and that the problems of using this estimation method are not so serious.

Table 6 reports the estimation results. The table shows the estimated parameter values for four constants (one for each currency) and for *ex post* real return, risk, trade shares and debt shares. It should be noted that $\overline{R}^2$ can be used here, as the variances of currency shares are relatively similar as indicated by the upper line in Table 7 that shows the diagonal elements of S, the residual variance-covariance matrix.

From Table 6 we see that both estimation techniques give practically the same parameter values, as should be expected since OLS is still unbiased. The explanatory power and the standard errors of the models are also the same.

Table 6. Estimation of currency demand

Ex post real returns 1980-1993

	b_{USD}	b_{DEM}	b_{GBP}	b_{JPY}	b_1	b_2	b_3	b_4	$\overline{R}^2$	*SE*
OLS	-0.286**	0.009	-0.072**	0.059*	-0.092	-0.096	0.868**	0.679**	0.949	0.044
t-values	-3.101	0.302	-2.732	2.232	-1.255	-0.813	5.525	4.196		
GLS	-0.249**	0.014	-0.077**	0.055*	-0.073	-0.032	0.856**	0.600**	0.948	0.044
t-values	-2.672	0.444	-2.910	2.055	-0.984	-0.267	5.396	3.673		

Note: * (**) indicates statistical significance at the 5 per cent (1 per cent) level. $\overline{R}^2$ is the coefficient of determination, corrected for degrees of freedom. *SE* is the standard error of the equation.

The constants for the dollar, pound and yen are all significant from zero. They also seem quite different from each other, indicating some specific currency affects. The coefficient on real return has a negative sign, but is not significant from zero at the 1 per cent level. The coefficient on risk is also negative, as should be expected, but it is also insignificant. The coefficients on trade and debt shares both have, however, correct signs and are well significant.

Table 7 shows the estimated variance-covariance matrix. As mentioned above volatility in currency shares seem to be quite similar. Correlations are however different, but all negative. These results indicate that it is more appropriate to use GLS than OLS.

Table 7. Estimated variances in S and correlations

	USD	DEM	GBP	JPY
$\hat{S}_{ii}$	0.028	0.021	0.015	0.022
USD	1.000			
DEM	-0.473	1.000		
GBP	-0.341	-0.050	1.000	
JPY	-0.441	-0.129	-0.063	1.000

These results strongly suggest that the transaction cost model has more explanatory power than the mean-variance model in explaining relative currency demand of the Central Bank of Iceland in the period 1980 to 1993. The Central Bank seems to have based its currency composition on the relative shares of currencies in the country's foreign trade and its foreign liabilities rather than on portfolio optimisation. These results are similar to those of Dooley (1987), Heller and Knight (1978) and Dooley, Lizondo and Matthieson (1989), who find that foreign trade, foreign debt (for developing countries) and exchange rate arrangements have significant explanatory power in explaining central banks' currency compositions.[26]

Finally, in Table 8 we show the estimated model for each currency. In these estimations there are only 14 observations, or 9 degrees of freedom, so one should be careful in interpreting the results.

Table 8. Estimation of currency demand for each currency

Ex post real return 1980-1993

w_i	Constant	b_{i1}	b_{i2}	b_{i3}	b_{i4}	$\overline{R}^2$	*SE*	DW	*F*
USD	-0.459	-0.409	0.511	1.359**	0.431	0.745	0.059	1.273	10.488
t-values	-1.776	-1.152	0.726	3.324	1.028				(0.001)
DEM	0.008	-0.057	-0.222	1.064	0.635	0.440	0.048	1.131	3.553
t-values	0.091	-0.341	-0.559	1.168	1.548				(0.053)
GBP	-0.062	-0.081	0.064	1.091*	-0.544	0.187	0.017	1.878	1.749
t-values	-1.324	-1.440	0.706	2.229	-1.742				(0.223)
JPY	0.053	-0.071	-0.092	-0.722	1.151**	0.635	0.033	1.692	6.656
t-values	1.524	-0.702	-0.706	-1.534	4.889				(0.009)

Note: * (**) indicates statistical significance at the 5 per cent (1 per cent) level. The *F*-test is a test for the significance of each equation (*p*-values in parenthesis). DW is the Durbin-Watson first order serial correlation test statistic.

The equations for the dollar and the yen have the largest explanatory power, while the equations for the pound and the mark have very little. The hypothesis that all parameters (except the constant) are non-significant cannot be rejected for the two latter currencies. In the dollar equation, the dollar share in foreign trade is significant at the 1 per cent level. The pound share in foreign trade is also significant at the 5 per cent level in the pound equation, but only marginally. In the yen equation, the yen share in foreign debt is significant at the 1 per cent level. In the mark equation there are, however, no significant parameters.

This indicates that the dollar share in the reserves can be explained largely by the dollar's share in foreign trade. The same applies to the pound, but to lesser extent.

[26]In all these studies the country composition of foreign trade is used as a proxy for the currency composition of foreign trade.

The yen share in the reserves can, however, be explained by the yen's share in foreign debt. This again indicates that the significance of foreign trade in the pooling model can be explained by the dollar and, to some extent, the pound, and that the significance of foreign debt can be explained by the yen.

4.2. Estimating the Inverse Demand Functions

Even though the mean-variance model seems strongly rejected by the data it is interesting to test formally whether restrictions imposed on the data by the mean-variance model can be rejected. Such a test can be derived from the model in previous section.[27]

Equation (4) can be written as an "inverse-demand function":

$$(11) \qquad E_t(X_t) = C + rW_tW_t,$$

where C is a vector of constants.[28] According to (11), for a given asset, it's expected excess rate of return in equilibrium would be higher if the share of that asset in the portfolio increased.

Frankel (1982) points out that if expectations are rational, the variance-covariance matrix of the forecast errors, $e_t \equiv X_t - E_t(X_t)$, is W_t. That is:

$$(12) \qquad E(e_t|Q_t) = 0; \quad E(e_te_t'|Q_t) = W_t.$$

This provides us with a simple test of the mean-variance model, by simply regressing *ex post* excessive real returns, X_t, on W_t. That should give an estimation of the variance-covariance matrix of residuals which should be proportional to the coefficient matrix, with the constant of proportionality equal to r. Therefore we can estimate the following model:

$$(13) \qquad X_t = C + B_tW_t + e_t,$$

and test the restrictions:

[27]It was Frankel (1982) who first suggested this approach. See also Engel and Rodrigues (1989, 1993) and Dumas (1994).

[28]In general C is not a constant but related to W_t (see equation (4)) but we assume that W_t is relatively constant through time and is therefore treated as a constant. Another reason for this assumption is that no constraints between C and the vector of errors can be imposed. See, for example, Engel and Rodrigues (1989 and 1993).

(14) $B_t = rW_t$.

Usually it is assumed that the coefficient matrix, B, is constant over time, i.e. that W is constant.[29] Recently the model has been extended to allow B to vary over time, for example according to an ARCH process.[30] Here it is assumed, however, that W is constant and B is therefore treated as a matrix of fixed coefficients.[31]

The null hypothesis is $b_{ij} = r s_{ij}$, where b_{ij} and s_{ij} are the $\{ij\}$ elements of B and W. A rejection of the null indicates that relative currency demand by the Central Bank of Iceland cannot be described by the mean-variance model. Here, excess returns are calculated using the dollar as a numeraire.[32]

Table 9 shows the results based on *ex post* real returns for the unrestricted asset demand functions. None of the parameter estimates are significant from zero at the 1 per cent level. A Wald statistic for the null hypothesis in (14) is $c^2(8)$ = 4.02 [p = 0.26], which indicates that the null cannot be rejected. At first sight this might indicate a positive result for the mean-variance model but this is not the case. The reason is that all the estimated parameters are non-significantly different from zero and therefore we cannot reject the hypothesis that all the r's are equal.

Table 9. Inverse demand functions using ex post real returns

Quarterly data 1981:1 - 1993:4

Table 9.*a*. Unrestricted asset demand functions

Dependent variable	C	w_{DEM}	w_{GBP}	w_{JPY}	R^2
x_{DEM}	-0.001	0.383	-1.295	-0.270	0.021
t-values	-0.008	0.794	-0.809	-0.428	
x_{GBP}	-0.168	0.762	-1.322	0.348	0.061
t-values	-1.162	1.532	-0.799	0.533	
x_{JPY}	-0.055	0.642	-2.229	0.136	0.069
t-values	-0.397	1.359	-1.420	0.220	

Note: * (**) indicates statistical significance at the 5 per cent (1 per cent) level.

Table 9.*b*. Variance-covariance matrix of residuals

	DEM	GBP	JPY
DEM	0.05748	0.04303	0.03946
GBP		0.05846	0.03071
JPY			0.05268

[29] See, for example, Frankel (1982).

[30] See Engel and Rodrigues (1989 and 1993).

[31] See, however, the discussion in section 2.3.

[32] See, for example, Dellas and Yoo (1991) and Engel and Rodrigues (1989, 1993). We also estimated the model using the minimum variance portfolio shares to calculate an alternative numeraire. The results from that model were almost identical to those reported here.

To get an estimate of r the restrictions $B = rW$ were imposed on the model. The resulting parameter estimate is $\hat{r} = 4.0$ (*t*-value = 0.616). This could indicate that the Central Bank of Iceland is relatively risk averse in its portfolio management. However, the standard error is very large, so a large array of parameter values cannot be ruled out. In particular, the value of zero is not rejected which is in accordance with the result that all the parameter estimates are non-significant.

The results therefore strongly suggest that the mean-variance model cannot explain the relative demand for currencies by the Central Bank of Iceland. This is a similar result to that found in other studies using the same approach.[33]

Engel and Rodrigues (1989) point out that a more appropriate approach is to allow W to vary over time. Their results show that this does not change the main conclusion, that the mean-variance model is rejected by the data.[34] They also try to explain this failure by allowing for measurement errors but to no success.

Another possible explanation for these negative results might be the assumption concerning the information set available to the Central Bank, Q_t. We have assumed that Q_t contains all information relevant available to the Bank at time *t* when the Bank makes its portfolio decisions. This assumption is sometimes called the strong form of the rational expectations hypothesis or Muth-rationality. As we are testing the mean-variance model and the rational expectations hypothesis simultaneously, this strong form of the rational expectations hypothesis might be rejected by the data, leading to a failure of the model. A weaker form of the rational expectations would be to assume that the Central Bank has a different information set, F_t, for which $F_t \subset Q_t$ and $E(e_t | F_t) = 0$, so that F_t is a subset of the complete information set and is orthogonal on the forecast errors.

As was described in the previous section, we estimated expected returns on currencies assuming that exchange rates follow a random walk process, which means that only future import prices need to be forecasted. We calculated the expected returns by estimating an ARIMA model for import prices, assuming that $F_t = \{\ln P_{t-j}\}_{j=0}^{\infty}$. Using estimations of expected excess returns, Table 10 shows the estimates of the inverse demand functions.[35]

The parameter estimation changes somewhat and now some of them are statistically significant. The relation between currency demands and returns is, however, mixed. The currency share of the yen is always significant, but has a wrong sign in its own demand equation. The Wald statistic for the null hypothesis is now

[33]See, for example, Dellas and Yoo (1991).

[34]They allow W_t to vary over time according to an ARCH model and as a function of market uncertainty, measured as unforeseen changes in money aggregates and oil prices.

[35]Dellas and Yoo (1991), using slightly different arguments, also test the mean-variance model assuming that exchange rates follow a random walk process.

$c^2(8) = 10.57$ [$p = 0.01$], so the null can now be rejected at the 1 per cent level. Table 10.*b* shows how the variances and covariances of the residuals have drastically reduced. This should be expected as the volatility of exchange rate returns contributes by far the largest to the volatility of real returns.

Table 10. Inverse demand functions using ex ante real returns

Quarterly data 1981:1 - 1993:4

Table 10.*a*. Unrestricted asset demand functions

Dependent variable	C	w_{DEM}	w_{GBP}	w_{JPY}	R^2
x_{DEM}	-0.024**	-0.026	-0.050	-0.093*	0.095
t-values	-2.497	-0.795	-0.456	-2.141	
x_{GBP}	0.046**	-0.048	-0.080	-0.277**	0.417
t-values	4.341	-1.299	-0.652	-5.755	
x_{JPY}	0.001	0.027	0.137	-0.414**	0.551
t-values	0.066	0.614	0.919	-7.075	

Note: * (**) indicates statistical significance at the 5 per cent (1 per cent) level.

Table 10.*b*. Variance-covariance matrix of residuals

	DEM	GBP	JPY
DEM	0.00026	0.00019	0.00027
GBP		0.00032	0.00030
JPY			0.00047

When the model is estimated again with the imposed restrictions $B = rW$, the estimated risk aversion is $\hat{r} = -405.3$ (*t*-value = -4.23) and the parameter value is significant from zero. The estimated risk aversion is very large and negative, which would be of serious concern had the restriction not been rejected by the data.

It seems, therefore, that the mean-variance model can be rejected as an explanation for relative currency demand of the Central Bank of Iceland, which is in accordance with the results of the previous model and a comparison of actual currency shares and those predicted by the mean-variance model.

5. Conclusions

During the last few years central banks have increasingly adopted the mean-variance approach when deciding the composition of their foreign reserves. Here we use the mean-variance model to compose the optimal currency shares for the Central Bank of Iceland for the period 1987 to 1993, assuming that the Bank is preserving the real value of reserves measured by currency weighted imports. Currency shares are calculated using both *ex post* and *ex ante* real returns, calculated from a forecasting model. The resulting currency demands are somewhat different from the actual demands in that period, indicating that the mean-variance model is not sufficient to explain the investment behaviour of the Central Bank of Iceland.

An alternative model for explaining relative currency demand, is the transaction cost model. Although we feel that the mean-variance model is a closer approximation to optimal investment behaviour of central banks, formal tests strongly reject the mean-variance model but show strong explanatory power of the transaction cost model. The poor results of the mean-variance model are further confirmed by a direct estimation of relative currency demands, which clearly rejects the model.

Appendix A. The Data

Reserves

Total reserves with gold which, in Iceland's case, has a small and stable share in total reserves. *Source*: Central Bank of Iceland and IMF: *Annual Report*, various publications.

Import prices

Weighted consumer prices of the 15 most important trading partners of Iceland, according to $P_t = \prod_{i=1}^{15} (e_{it} P_{it}^{*})^{a_{it}}$, where e_i is the nominal exchange rate between the Icelandic krona and currency *i* (i.e. krona per currency *i*), P_i^{*} is the consumer price index in country *i* and a_i is the weight of currency *i* in trade transactions (calculated as the average of currency *i* purchases and sales by domestic commercial banks). *Source*: Central Bank of Iceland and IMF: *International Financial Statistics*.

Return on currencies

Real return on various currencies in the Eurodeposit market, using three month averages. Libor rates for three months deposits in each currency (for GBP the Paris interbank rate is used). Real return is calculated as:

$$(1+m_{it}) = \frac{(1+r_{it})(e_{it+1}/e_{it})}{(P_{t+1}/P_t)},$$

where m_i is the *ex post* real return on currency *i*, r_i is the nominal return on currency *i* and *P* is the import price index as defined above. All returns are annual percentages. *Source*: IMF: *International Financial Statistics* and the authors calculations.

Volatility of returns

Standard deviations of real returns on the Eurodeposit market within each period, calculated from monthly returns of each currency. *Source*: Authors calculations.

Appendix B. The Optimal Portfolio Composition

The most simple way to solve the model in the main text is to represent the problem in matrix form. The variance of the reserves can then be written as:

$$\text{(B.1)} \quad s_{Rt}^{2} = W_t' W_t W_t,$$

where $W_t = (w_{1t}, \ldots, w_{Gt})'$ is a vector of currency shares and W_t is a (*G*×*G*) variance-covariance matrix of currency returns.

Equation (B.1) is minimised with respect to:

(B.2) $m_{Rt} = W_t' M_t; \quad W_t' c = 1; \qquad W_t \geq O,$

where $M_t = (m_{1t}, \ldots, m_{Gt})'$ is a vector of expected currency returns, c is a (G×1) unit vector and O is a (G×1) vector of zeros.

The solution to this problem gives a composition of reserves that lies on the efficient frontier for a given return:[36]

(B.3) $W_t = W_t^{-1}(M_t, c) A_t^{-1}(m_{Rt}, 1)',$

where A_t is defined as:

$$A_t = \begin{pmatrix} M_t' \Omega_t^{-1} M_t & M_t' \Omega_t^{-1} c \\ M_t' \Omega_t^{-1} c & c' \Omega_t^{-1} c \end{pmatrix}.$$

The solution for s_{Rt}^2 which lies on the frontier, s_{Rt}^{2*}, is:

(B.4) $s_{Rt}^{2*} = (m_{Rt}, 1) A_t^{-1} (m_{Rt}, 1)'.$

By minimising (B.4) with respect to m_{Rt}, one gets the minimum variance portfolio:

(B.5) $W_t^* = W_t^{-1} c \big/ c' W_t^{-1} c.$

If the central bank is willing to increase the return on the reserves by taking more risk, a new portfolio can be obtained by maximising an expected utility function, such as equation (3) in the main text, with respect to m_{Rt} and given equation (B.4):

(B.6) $W_t^{**} = W_t^{-1} c \big/ c' W_t^{-1} c + r^{-1} W_t^{-1} (M_t - c(M_t' W_t^{-1} c \big/ c' W_t^{-1} c))$

or:

(B.7) $W_t^{**} = W_t^* + r^{-1} W_t^{-1} (M_t - c m_{Rt}^*) = W_t^* + r^{-1} W_t^{-1} E_t(X_t).$

[36]See for example. Horii (1986).

where W_t^* is the minimum variance portfolio and m_{Rt}^* is the expected real return on the minimum variance portfolio. $E_t(X_t) \equiv M_t - cm_{Rt}^*$ is therefore a vector of expected real returns in excess of the return of the minimum variance portfolio.

For currency *k* the solution is therefore:

$$\text{(B.8)} \quad w_{kt}^{**} = \left(\frac{\sum_i^G q_{kit}}{\sum_i^G \sum_j^G q_{ijt}} \right) + r^{-1} \left\{ \sum_{j=1}^{G} q_{kjt} \left(m_{jt} - \frac{\sum_i^G \sum_j^G q_{ijt} m_{it}}{\sum_i^G \sum_j^G q_{ijt}} \right) \right\},$$

where $\{q_{ijt}\} \equiv \{W_{ijt}^{-1}\}$.

Appendix C. Expected Real Return on Currencies

Expected return on currency *i* can be written as (in logs for convenience):

$$\text{(C.1)} \quad m_{it} = r_{it} + (E_t(\ln e_{it+1}) - \ln e_{it}) - (E_t(\ln P_{t+1}) - \ln P_t),$$

To calculate the expected real return on currency *i*, two forecasts are needed. One is a forecast of the future exchange rate between the domestic currency and currency *i* and the other is a forecast of country *i*'s import price index.

To forecast the exchange rate it is assumed that the return on currencies follows a no-arbitrage condition:

$$\text{(C.2)} \quad \ln e_{it+1} = \ln e_{it} + u_{it+1},$$

where u_{it+1} is a forecast error which is distributed as $(0, s_u^2)$. If it is assumed that $E_t(\ln e_{it+1}) < \infty$, the stochastic process for the exchange rate is a Martingale process. A stronger assumption would be to assume that u_{it+1} is an i.i.d. innovation, which produces a random walk process.

Recent research in short term exchange rates indicates that exchange rates are not stationary but that returns on currencies are. A unit root test for the exchange rate of the krona per foreign currencies indicates that the exchange rate does indeed contain a unit root.

Table C.1. Unit root tests of the krona versus various currencies[a]

Monthly data: 1980:6-1994:6

	USD	DEM	GBP	JPY
ADF(4) without trend	-3.603	-2.602	-3.077	-2.820
ADF(4) with trend	-2.194	-1.218	-1.540	-1.843

[a] The null hypothesis is that the series in question is $I(1)$, i.e. that $b = 1$in $\ln e_{it+1} = b \ln e_{it} + u_{it+1}$, where u_{it+1} is an i.i.d. innovation. The table shows the Augmented Dickey-Fuller statistic, t, for the estimation of the model $D\ln e_{it+1} = a - (1-b)\ln e_{it} + \sum_{j=0}^{3} g_j D\ln e_{it-j}$. The rejection region is $\{ t \in \Re | t < c \}$, with c = -3.47, -2.88 and -2.58 at a significant level of 1, 5 and 10 per cent respectively, for ADF(4) without trend, and -4.01, -3.44 and -3.19 for a significant level of 1, 5 and 10 per cent for ADF(4) with trend. See MacKinnon (1991).

The model in (C.2) indicates that all information on future developments of currencies is already contained in the current spot rate. All unexpected events are therefore contained in the forecast error, u_{it+1}, and the best forecast for the future spot rate is the current one:

$$\text{(C.3)} \quad E_t(\ln e_{it+1}) = \ln e_{it}.$$

Based on these properties of exchange rates, expected real returns are only based on the forecast of future development of the import price level:

$$\text{(C.4)} \quad m_{it} = r_{it} - (E_t(\ln P^*_{t+1}) - \ln P^*_t),$$

where $P^*_t = \prod_{i=1}^{N} (P^*_{it})^{a_{it}}$. For forecasting the import price index an ARIMA model is developed. The price index is an $I(1)$ process.[37] By analysing the autocorrelation function of $D\ln P^*_{it}$ and the Akaike and Schwartz criteria, an ARIMA(1,1,0) model was chosen as the most suitable description of the log-differenced import price level for the period 1980:1 to 1986:12. The estimated model is (t-values in parentheses):

$$\text{(C.5)} \quad \begin{array}{l} D\ln P^*_{t+1} = 0.004 + 0.785\, D\ln P^*_t \\ \qquad\qquad\quad (5.4) \quad\; (13.1) \end{array}$$

$\overline{R}^2$ = 0.679, SE = 0.002, $F(1,80)$ = 172.489 (p = 0.00), BG(12) = 11.665 (p = 0.47), JB(2) = 0.023 (p = 0.99), WHITE(2) = 1.245 (p = 0.54), CHOW(84,164) = 0.663 (p = 0.97), Theil's U = 0.175

[37] The Augmented Dickey-Fuller test statistic gives -3.14, with a 1 per cent critical value of -3.47, indicating the null hypothesis of a unit root in the import price level cannot be rejected.

The model seems to fit the series quite well. The Breusch-Godfrey serial correlation test, Bera-Jarque normality test and White's heteroscedasticity test indicate that the residual is a normally distributed white noise process. A Chow forecast test for the period 1987:1 to 1993:12 indicates no changes in the parameter vector. The Theil's U statistic indicates that the model provides a better out of sample forecast of the log-differenced import price level for the period 1987:1 to 1993:12 than a simple random walk model.

From equation (C.5) the expected import price level can finally be calculated as:

$$E_t(\ln P_{t+1}^{*}) = 0.004 + 1.785 \ln P_t^{*} - 0.785 \ln P_{t-1}^{*}, \tag{C.6}$$

which is the equation used to calculated expected real return for the period 1987:1 to 1993:12 in the main text.

References

Allen, W. (1994), "Current Issues in Reserve Management", *Economic and Financial Review*, 1.

Ben-Bassat, A. (1980), "The Optimal Composition of Foreign Exchange Reserves", *Journal of International Economics*, 10.

Bollerslev, T., R. Engle and D. Nelson (1993), "ARCH Models", article prepared for the forthcoming book: *The Handbook of Econometrics*, vol. 4, North Holland.

Dellas, H. and C. Yoo (1991), "Reserve Currency Preferences of Central Banks: The Case of Korea", *Journal of International Money and Finance*, 10.

Dooley, M. (1987), "An Analysis of the Management of the Currency Composition of Reserve Assets and External Liabilities of Developing Countries", *The Reconstruction of International Monetary Arrangements*, editor, R. Aliber. Macmillan.

Dooley, M., J. Lizondo and D. Mathieson (1989), "The Currency Composition of Foreign Exchange Reserves", *IMF Staff Papers*, 36.

Downes, P. (1989), "Managing Foreign Exchange Reserves", *Finance and Development*, 1.

Dumas, B. (1994), "International Capital Markets", *The Handbook of International Macroeconomics*, editor F. van der Ploeg. Blackwell.

Engel, C. and A. Rodrigues (1989), "Tests of International CAPM with Time-Varying Covariances", *Journal of Applied Econometrics*, 4.

Engel, C. and A. Rodrigues (1993), "Tests of Mean-Variance Efficiency of International Equity Markets", *Oxford Economic Papers*, 45.

Engle, R. (1982), "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation", *Econometrica*, 50.

Frankel, J. (1982), "In Search of the Exchange Risk Premium: A Six Currency Test Assuming Mean-Variance Optimisation", *Journal of International Money and Finance*, 1.

Frenkel, J. and B. Jovanovic (1981), "Optimal International Reserves: A Stochastic Framework", *Economic Journal*, 91.

Heller, H. and M. Kahn (1978), "The Demand for Reserves under Fixed and Floating Exchange Rates", *IMF Staff Papers*, 25.

Heller, H. and M. Knight (1978), "Reserve-Currency Preferences of Central Banks", *Princeton Essays in International Finance*, no. 131.

Horii, A. (1986), "The Evolution of Reserve Currency Diversification", *BIS Economic Papers*, no. 18.

International Money Fund, *Annual Report*, various years.

Landell-Mills, J. (1989), "The Demand for International Reserves and their Opportunity Cost", *IMF Staff Papers*, 36.

Lehto, T. (1994), "The Level of a Central Bank's International Reserves: Theory and Cross-Country Analysis", *Bank of Finland Discussion Paper*, no. 15/94.

Levy, H. and M. Sarnat (1975), "Devaluation Risk and the Portfolio of International Investment", *International Capital Markets*, editors E. Elton and M. Gruber. North-Holland.

Lizondo, J. and D. Mathieson (1987), "The Stability of the Demand for International Reserves", *Journal of International Money and Finance*, 6.

MacKinnon, J. (1991), "Critical Values for Cointegration Tests", *Long-Run Economics Relationships*, editors R. Engle and C. Granger. Oxford University Press.

Markowitz, H. (1952), "Portfolio Selection", *Journal of Finance*, 7.

Papaioannou, M. and T. Temel (1993), "Portfolio Performance of the SDR and Reserve Currencies: Tests using the ARCH Methodology", *IMF Working Papers*, 39/10.

Pétursson, T. and H. Tómasson (1994), "Small National Financial Markets in Transition: The Case of Iceland", Institute of Economic Studies, University of Iceland, no. 2/1994.

Rikkonen, K. (1989), "The Optimal Currency Distribution of a Central Bank's Foreign Exchange Reserves", *Bank of Finland Discussion Paper*, no. 28/89.

Roger, S. (1993), "The Management of Foreign Exchange Reserves", *BIS Economic Papers*, no. 38.

Sargent, T. (1987), *Macroeconomic Theory*, 2nd edition. Academic Press.

Tobin, J. (1958), "Liquidity Preference as Behaviour Towards Risk", *Review of Economic Studies*, 26.

de Vries, C. (1994), "Stylised Facts of Nominal Exchange Rate Returns", *The Handbook of International Macroeconomics*, editor F. van der Ploeg. Blackwell.